RESEARCH METHODOLOGY-SCIENCE (2018)

1. What is the next term in the following sequence ?

7, 11, 13, 17, 19, 23, 29, ...

(A) 37 (B) 35

(C) 31 (D) 33

2. Which of the following numbers is a perfect square ?

(A) 1022121 (B) 2042122

(C) 3063126 (D) 4083128

3. If 42 $\rightarrow$ 26, 71 $\rightarrow$ 78, 33 $\rightarrow$ 16, then 62 $\rightarrow$

(A) 68 (B) 54

(C) 38 (D) 39

4. Approximately how much blood flows per day through a normal human heart beating 70 times per minute, having a relaxed volume of 110 cc and compressed volume of 70 cc ?

(A) 7150 litres (B) 4000 litres

(C) 28000 litres (D) 11100 litres

5. Find the missing word : A, AB, _____, ABBABAAB

(A) AABB (B) ABAB

(C) ABBA (D) BAAB

6. How many digits are there in 3^{16} when it is expressed in the decimal form ?

(A) Three (B) Six

(C) Seven (D) Eight

7. The sum of the first n natural numbers with one of them missed is 42. What is the number that was missed ?

(A) 1 (B) 2

(C) 3 (D) 4

8. If N, E and T are distinct positive integers such that $N \times E \times T = 2013$, then which of the following is the maximum possible sum of N, E and T ?

(A) 39 (B) 2015

(C) 671 (D) 675

9. Two plane mirrors facing each other are kept at 60° to each other. A point is located on the angle bisector. The number of images of the point is :

(A) 6 (B) 3

(C) 5 (D) infinite

10. A 3 m long car goes past a 4 m long truck at rest on the road. The speed of the car is 7 m/s. The time taken to go past is :

(A) 4/7 s (B) 1 s

(C) 7/4 s (D) 10/7 s

11. The dimensions of a floor are 18×24. What is the smallest number of identical square tiles that will pave the entire floor without the need to break any tile ?

(A) 6 (B) 24

(C) 12 (D) 8

12. I bought a shirt at l0% discount and sold it to a friend at a loss of l0%. If the friend paid me Rs. 729.00 for the shirt, what was the undiscounted price of the shirt ?

(A) Rs. 900 (B) Rs. 800

(C) Rs. 1000 (D) Rs. 911.25

13. How many non-negative integers less than 10,000 are there such that the sum of the digits of the number is divisible by three ?

(A) 1112 (B) 2213

(C) 2223 (D) 3334

14. 20 teachers of a school either teach mathematics or physics. 12 of them teach mathematics while 4 teach both the subjects. Then the number of teachers teaching physics only is :

(A) 8 (B) 12

(C) 16 (D) None of these

15. Let A, B be the ends of the longest diagonal of the unit cube. The length of the shortest path from A to B along the surface is :

(A) $\sqrt{3}$ (B) $1+\sqrt{2}$

(C) $\sqrt{5}$ (D) 3

16. In an examination, a student scores 4 marks for every correct answer and loses 1 mark for every wrong answer. If he attempts in all 60 questions and secures 130 marks, the number of questions he attempts correctly, is :

(A) 35 (B) 38

(C) 40 (D) 42

17. It takes 2 hours for Tiwari and Deo to do a job. Tiwari and Hari take 3 hours to do the same job. Deo and Hari take 6 hours to do the same job. Which of the following statements is *incorrect* ?

(A) Tiwari alone can do the job in 3 hours

(B) Deo alone can do the job in 6 hours

(C) Hari does not work at all

(D) Hari is the fastest worker

18. If you count 21 letters in the English alphabet from the end and 20 letters from the beginning, which letter will appear exactly in the middle of the sequence thus formed ?

(A) M (B) L

(C) K (D) N

19. A train running at 36 km/h crosses a mark on the platform in 8 sec and takes 20 sec to cross the platform. What is the length of the platform ?

(A) 120 m (B) 280 m

(C) 40 m (D) 160 m

20. Water is slowly dripping out of a tiny hole at the bottom of a hollow metallic sphere initially full of water. Ignoring the water that has flowed away, the centre of mass of the system :

(A) remains fixed at the centre of the sphere

(B) moves down steadily as the amount of water decreases

(C) moves down for some time but eventually returns to the centre of the sphere

(D) moves down until half of the water is lost and then moves up

21. The missing number is :

3	6	8
5	8	4
4	7	?

(A) 6 (B) 7

(C) 8 (D) 9

22. Reena is twice as old as Sunita. Three years ago, she was three times as old as Sunita. How old is Reena now ?

(A) 6 years

(B) 7 years

(C) 12 years

(D) 8 years

23. A bicycle tube has a mean circumference of 200 cm and a circular cross-section of diameter 6 cm. What is the approximate volume of water (in cc) required to completely fill the tube, assuming that it does not expand ?

(A) 600 π (B) 1200 π

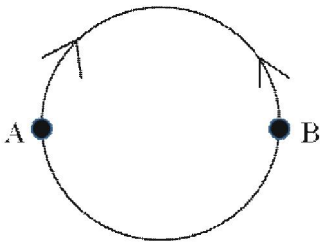
(C) 3600 π (D) 1800 π

24. Six persons P, Q, R, S, T and U are standing in a circle. Q is between S and R. P is between T and R. U is to the right of S. Who is between P and U ?

(A) Q (B) R

(C) T (D) S

25.



Two ants, initially at diametrically opposite points A and B on a circular ring of radius R, start crawling towards each other. The speed of the one at A is half of that of the one at B. The point at which they meet is at a straight line distance of :

(A) R from A

(B) $\frac{3R}{2}$ from A

(C) R from B

(D) $\frac{3R}{2}$ from B

26. A person completely under sea water tracks the Sun. Compared to an observer above water, which of the following observations would be made by the underwater observer ?

(A) Neither the time of sunrise or sunset nor the angular span of the horizon changes.

(B) Sunrise is delayed, sunset is advanced, but there is no change in the angular span of the horizon.

(C) Sunrise and sunset times remain unchanged, but the angular span of the horizon shrinks.

(D) The duration of the day and the angular span of the horizon, both decrease.

27. The number of boys in a class is three times the number of girls. Which one of the following numbers cannot represent the total number of children in the class ?

(A) 48

(B) 42

(C) 44

(D) 40

28. Two coconuts have spherical space inside their kernels, with the first having an inner diameter twice that of the other. The larger one is half filled with liquid, while the smaller is completely filled. Which of the following statements is *correct* ?

(A) The larger coconut contains 4 times the liquid in the smaller one.

(B) The larger coconut contains twice the liquid in the smaller one.

(C) The coconuts contain equal volumes of liquid.

(D) The smaller coconut contains twice the liquid in the larger one.

29. A tiger usually stalks its prey from a direction that is upwind of the prey. The reason for this is :

(A) the wind aids its final burst for killing the prey

(B) the wind carries the scent of the prey to the tiger and helps the tiger locate the prey easily

(C) the upwind area usually has denser vegetation and better camouflage

(D) the upwind location aids the tiger by not letting its smell reach the prey

30. A cellphone tower radiates 1W power while the handset transmitter radiates 0.1 mW power. The correct comparison of the radiation energy received by your head from a tower 100 m away (E_1) and that from a handset held to your ear (E_2) is :

(A) $E_1 >> E_2$

(B) $E_2 >> E_1$

(C) $E_1 = E_2$ for communication to be established

(D) insufficient data even for a rough comparison

31. The pitch of a spring is 5 mm. The diameter of the spring is 1 cm. The spring spins about its axis with a speed of 2 rotations/s. The spring appears to be moving parallel to its axis with a speed of :

(A) 1 mm/s (B) 5 mm/s

(C) 6 mm/s (D) 10 mm/s

32. A boy holds one end of a rope of length l and the other end is fixed to a thin pole of radius r ($l >> r$). Keeping the rope taut, the boy goes around the pole causing the rope to get wound around the pole. Each round takes 10 s. What is the speed (in units of s^{-1}) with which the boy approaches the pole ?

(A) $\frac{\pi r}{5}$ (B) $\frac{\pi l}{5}$

(C) $20\pi(r+l)$ (D) $\frac{20\pi(r-l)}{5}$

33. A rectangular flask of length 11 cm, width 8 cm and height 20 cm has water filled up to height 5 cm. If 21 spherical marbles of radius 1 cm each are dropped in the flask, what would be the rise in water level ?

(A) 8.8 cm (B) 10 cm

(C) 1 cm (D) 0 cm

34. Deepak starts walking straight towards east. After walking 75 metres, he turns to the left and walks 25 metres straight. Again he turns to the left, walks a distance of 40 metres straight, again he turns to the left and walks a distance of 25 metres. How far is he from the starting point ?

(A) 25 m (B) 50 m

(C) 115 m (D) 35 m

35. A leaf appears green in daylight. If this leaf were observed in red light, what colour would it appear to have ?

(A) green (B) black-brown

(C) red (D) blue

36. The sum of two numbers is equal to sum of square of 11 and cube of 9. The larger number is $(5)^2$ less than square of 25. What is the value of the sum of twice of 24 per cent of the smaller number and half of the larger number ?

(A) 415 (B) 400

(C) 410 (D) 420

37. For which one of the following statements is the converse NOT true ?

(A) If a patient dies even with excellent medical care, he likely had terminal illness.

(B) If a person gets employed, he has good qualifications.

(C) If an integer is even, it is divisible by two.

(D) If an integer is odd, it is not divisible by two.

38. How many numbers from 1 to 100 are there each of which is not only exactly divisible by 4 but also has 4 as a digit ?

(A) 21 (B) 10

(C) 20 (D) 7

39. If a plant with green leaves is kept in a dark room with only green light ON, which one of the following would we observe ?

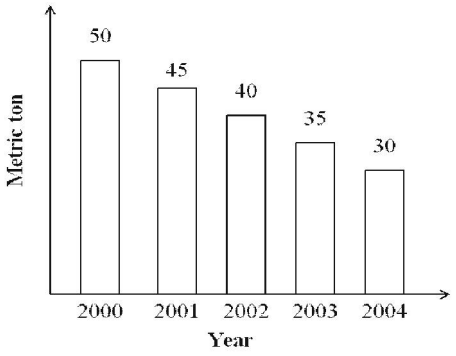
(A) The plant appears brighter than the surroundings

(B) The plant appears darker than the surroundings

(C) We cannot distinguish the plant from the surroundings

(D) It will have above normal photosynthetic activity

40. Wheat production of a country over a number of years is shown. Which year recorded highest per cent reduction in production over the previous year ?



(A) 2001

(B) 2002

(C) 2003

(D) 2004

41. L is the tallest and eldest of a group of five people K, L, M, N and P. M is elder to N and shorter than K. M and P are of same age and P is taller than K. N and K are of same height and K is younger to P. Which of the following inferences is certain ?

(A) P is taller than M

(B) N is the youngest

(C) N is elder to P

(D) N is elder to K

42. In a fast moving car with open windows, the driver feels a continuous incoming breeze. The pressure inside the car, however, does not keep increasing because :

(A) Air coming in from the front window goes out from the rear.

(B) Air comes in as well as goes out through every window but the driver only feels the incoming one.

(C) No air actually comes in and the feeling of breeze is an illusion.

(D) Cool air reduces the temperature therefore the pressure does not increase.

43. A tall metal cylinder is filled end-to-end with n snugly fitting spherical wax balls of diameter d. If the balls melt completely, the volume fraction occupied by the melted wax is :

(A) independent of both d and n

(B) dependent on both d and n

(C) independent of d, but dependent on n

(D) dependent on d, but independent of n

44. In each of the following groups of words is a hidden number, based on which you should arrange them in descending order. Pick the *correct* answer :

E. Papers I Xeroxed
F. Wi-Fi veteran
G. Yourself ourselves
H. Breaks even

(A) E, F, G, H
(B) E, G, F, H
(C) H, F, G, E
(D) H, E, F, G

45. Four circles of unit radius each are drawn such that each one touches two others and their centres lie on the vertices of a square. The area of the region enclosed between the circles is :

(A) π-1
(B) π-2
(C) 4-π
(D) 3-π

46. A film projector and microscope give equal magnification. But a film projector is not used to see living cells because :

(A) a living cell cannot be placed in a film projector.
(B) the viewer's eye is close to a microscope whereas it is far away from the projector's screen.
(C) a microscope produces a virtual image whereas a projector produces a real image.
(D) a microscope has greater resolving power than a projector.

47. Comparing numerical values, which of the following is different from the rest ?

(A) The ratio of the circumference of a circle to its diameter.
(B) The sum of the three angles of a plane triangle expressed in radians.
(C) 22/7.
(D) The net volume of a hemisphere of unit radius, and a cone of unit radius and unit height.

48. Seeds when soaked in water gain about 20% by weight and 10% by volume. By what factor does the density increase ?

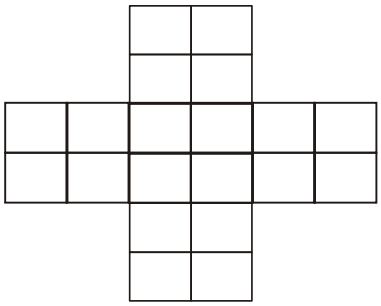
(A) 1.20

(B) 1.10

(C) 1.11

(D) 1.09

49.



The number of squares in the above figure is :

(A) 30 (B) 29

(C) 25 (D) 20

50. Five persons A, B, C, D, and E are sitting in a row with C in the middle of the group. If D is at an extreme end and there are at least two persons between B and E, then which of the following statements is *incorrect* ?

(A) E can be on extreme left

(B) E can be on extreme right

(C) A cannot be on extreme left

(D) A is always a neighbour of B or D

MATHEMATICS

51. Let $f_n(x)=(-x)^n$, $x\in[0,1]$. Then decide which of the following is *true* ?

(A) There exists a pointwise convergent subsequence of f_n.

(B) f_n has no pointwise convergent subsequence.

(C) f_n converges pointwise everywhere.

(D) f_n has exactly one pointwise convergent subsequence.

52. Let $f : \mathrm{R} \to \mathrm{R}$ be defined by :

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Then :

(A) f is not continuous

(B) f is continuous but not differentiable

(C) f is differentiable

(D) f is not bounded

53. Let $\mathrm{S}=\{x\in[-1,4] : \sin(x)>0\}$. Which of the following is *true* ?

(A) inf (S) < 0

(B) sup (S) does not exist

(C) sup (S) = π

(D) inf (S) = $\pi/2$

54. Which of the following is convergent ?

(A) $\sum_{n=1}^{\infty} n^2 2^{-n}$

(B) $\sum_{n=1}^{\infty} n^{-2} 2^n$

(C) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n \log(1+1/n)}$

55. Let k be a positive integer and let $S_k = \{x \in [0,1]$: a decimal expansion of x has a prime digit at its kth place$\}$. Then the Lebesgue measure of S_k is :

(A) 0 (B) 4/10

(C) $(4/10)^k$ (D) 1

56. Let (X, d) be a metric space. Then :

(A) An arbitrary open set G in X is a countable union of closed sets.

(B) An arbitrary open set G in X cannot be countable union of closed sets if X is connected.

(C) An arbitrary open set G in X is a countable union of closed sets only if X is countable.

(D) An arbitrary open set G in X is a countable union of closed sets only if X is locally compact.

57. Which of the following subsets of R^4 is a basis of R^4 ?

$B_1 = \{(1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1)\}$
$B_2 = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$
$B_3 = \{(1,2,0,0), (0,0,1,1), (2,1,0,0), (-5,5,0,0)\}$

(A) Only B_1 (B) B_1 and B_2 but not B_3

(C) B_1, B_2 and B_3 (D) B_1 and B_3 but not B_2

58. Let A and B be real invertible matrices such that AB = –BA. Then :

(A) Trace (A) = 1, Trace (B) = 0

(B) Trace (A) = Trace (B) = 1

(C) Trace (A) = 0, Trace (B) = 1

(D) Trace (A) = Trace (B) = 0

59. The matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is :

(A) non-negative definite but not positive definite.

(B) positive definite.

(C) negative definite.

(D) neither negative definite nor positive definite.

60. Let A be an $n \times m$ matrix with each entry equal to +1, –1 or 0 such that every column has exactly one +1 and exactly one –1. Then :

(A) Rank A $\leq n-1$ (B) Rank A = m

(C) $n \leq m$ (D) $n-1 \leq m$

61. Consider the matrix $A(x) = \begin{bmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{bmatrix}$; $x \in \mathrm{R}$. Then :

(A) A(x) has eigenvalue 0 for some $x \in \mathrm{R}$

(B) 0 is not an eigenvalue of A(x) for any $x \in \mathrm{R}$

(C) A(x) has eigenvalue 0 for all $x \in \mathrm{R}$

(D) A(x) is invertible for every $x \in \mathrm{R}$

62. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ \beta \end{bmatrix}$. Then the system AX = b over the real numbers has :

(A) no solution whenever $\beta \neq 7$

(B) an infinite number of solutions whenever $\alpha \neq 2$

(C) an infinite number of solutions if $\alpha = 2$ and $\beta \neq 7$

(D) a unique solution if $\alpha \neq 2$

63. The radius of convergence of the series $\sum_{n=1}^{\infty} z^{n^2}$:

(A) 0 (B) 8

(C) 1 (D) 2

64. Let C the circle $|z| = 3/2$ in the complex plane that is oriented in the counter clockwise direction. The value of a for which $\int_C \left(\frac{z+1}{z^2-3z+2} + \frac{a}{z-1} \right) dz = 0$:

(A) 1 (B) –1

(C) 2 (D) –2

65. Suppose f and g are entire functions and $g(z) \neq 0$ for all $z \in C$. If $|f(z)| \leq |g(z)|$, then :

(A) $f(z) \neq 0$ for all $z \in C$

(B) f is a constant function

(C) $f(0) = 0$

(D) for some $c \in C$, $f(z) = cg(z)$

66. Suppose f is holomorphic in an open neighbourhood of $z_0 \in C$. Given that the series $\sum_{n=0}^{\infty} f^{(n)}(z_0)$ converges absolutely, then :

(A) f is constant

(B) f is a polynomial

(C) f can be extended to an entire function

(D) $f(x) \in R$ for all $x \in R$

67. The function $f : C \to C$ defined by $f(z) = e^z + e^{-z}$ has :

(A) finitely many zeros

(B) no zeros

(C) only real zeros

(D) has infinitely many zeros

68. Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor divisible by 5. The number of elements in S is :

(A) 480 (B) 420

(C) 360 (D) 240

69. Let S_n denote the permutation group on n symbols and A_n be the subgroup of even permutations. Which of the following is *true* ?

(A) There exists a finite group which is not a subgroup of S_n for any $n \geq 1$.

(B) Every finite group is a subgroup of A_n for some $n \geq 1$.

(C) Every finite group is a quotient of A_n for some $n \geq 1$.

(D) No finite abelian group is a quotient of S_n for $n > 3$.

70. Let C be the field of complex numbers and C* be the group of non-zero complex numbers under multiplication. Then which of the following is *true* ?

(A) C* is cyclic

(B) Every finite subgroup of C* is cyclic

(C) C* has finitely many finite subgroups

(D) Every proper subgroup of C* is cyclic

71. Let R be a subring of Q containing 1. Then which of the following is necessarily *true* ?

(A) R is a principal ideal domain (PID)

(B) R contains infinitely many prime ideals

(C) R contains a prime ideal which is not a maximal ideal

(D) for every maximal ideal m in R, the residue field R/m is finite

72. What is the number of non-singular 3×3 matrices over F_2, the finite field with two elements ?

(A) 3^2 (B) 384

(C) 2^3 (D) 168

73. Let $z = e^{\frac{2\pi i}{7}}$ and let $\theta = z + z^2 + z^4$. Then :

(A) $\theta \in Q$

(B) $\theta \in Q(\sqrt{D})$ for some $D > 0$

(C) $\theta \in Q(\sqrt{D})$ for some $D < 0$

(D) $\theta \in iR$

74. Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to :

(A) the cyclic group of order 6

(B) the permutation group on {1, 2, 3}

(C) the permutation group on {1, 2, 3, 4, 5, 6}

(D) the permutation group on {1}

75. Let A be a connected open subset of R^2. The number of continuous surjective functions from $\bar{A}$ (the closure of A in R^2) to Q is :

(A) 1 (B) 0

(C) 2 (D) not finite

76. Let $f : R \to R$ be a continuous map. Choose the *correct* statement :

(A) f is bounded

(B) The image of f is an open subset of R

(C) $f(A)$ is bounded for all bounded subsets A of R.

(D) $f^{-1}(A)$ is compact for all compact subsets A of R.

77. Which of the following spaces is *not* separable ?

(A) R with the trivial topology (B) The Cantor set as a subspace of R

(C) R with the discrete topology (D) None of these

78. Let $(x(t), y(t))$ satisfy the system of ODEs $\frac{dx}{dt} = -x + ty; \frac{dy}{dt} = tx - y$. If $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ are two solutions and $\Phi(t) = x_1(t)y_2(t) - x_2(t)y_1(t)$, then $\frac{d\Phi}{dt}$ is equal to :

(A) 2Φ (B) -2Φ

(C) $-\Phi$ (D) Φ

79. Consider the differential equation $(x-1)y'' + xy' + \frac{1}{x}y = 0$. Then :

(A) $x = 1$ is the only singular point

(B) $x = 0$ is the only singular point

(C) both $x = 0$ and $x = 1$ are singular points

(D) neither $x = 0$ nor $x = 1$ are singular points

80. The set of real numbers λ for which the boundary value problem $\frac{d^2y}{dx^2}+\lambda y = 0,\ y(0) = 0,\ y(\pi) = 0$ has nontrivial solutions is :

(A) R

(B) $(-\infty, 0)$

(C) $\{\sqrt{n} : n \text{ is a positive integer}\}$

(D) $\{n^2 : n \text{ is a positive integer}\}$

81. Let $u(x, t)$ be the solution of the initial value problem $u_{tt} - u_{xx} = 0$, $u(x,0) = x^3$, $u_t(x,0) = \sin x$. Then $u(\pi, \pi)$ is :

(A) $4\pi^3$ (B) π^3

(C) 0 (D) 4

82. The PDE $x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y^2} = 0$ is :

(A) elliptic for $x < 0,\ y > 0$

(B) hyperbolic for $x > 0,\ y < 0$

(C) elliptic for $x > 0,\ y < 0$

(D) hyperbolic for $x > 0,\ y > 0$

83. The partial differential equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+u$ can be transformed to $\frac{\partial v}{\partial t}=\frac{\partial^2 v}{\partial x^2}$ for :

(A) $v = e^{-t}u$

(B) $v = e^{t}u$

(C) $v = tu$

(D) $v = -tu$

84. The iterative method $x_{n+1} = g(x_n)$ for the solution of $x^2 - x - 2 = 0$ converges quadratically in a neighbourhood of the root $x = 2$ if $g(x)$ equals :

(A) $x^2 - 2$ (B) $(x-2)^2 - 6$

(C) $1 + \frac{2}{x}$ (D) $\frac{x^2+2}{2x-1}$

85. Consider the initial value problem $\frac{dy}{dx} = x + y$, $y(0) = 1$. Then the approximate value of the solution $y(x)$ at $x = 0.2$, using improved Euler method, with $h = 0.2$ is :

(A) 1.24 (B) 1.20

(C) 1.11 (D) 1.48

86. The value of function $f(x)$ at 5 discrete points are given below :

x	0	0.1	0.2	0.3	0.4
$f(x)$	0	10	40	90	160

Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)\,dx$ is :

(A) 10.8 (B) 13.4

(C) 18.7 (D) 22.0

87. If $J[y] = \int_1^2 (y'^2 + 2yy' + y^2)\,dx$, $y(1) = 1$ and $y(2)$ is arbitrary, then the extremal is :

(A) e^{x-1} (B) e^{x+1}

(C) e^{1-x} (D) e^{-x-1}

88. The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3} dt$; $x(1)=3,\ x(2)=18$ $\left(\text{where } \dot{x} \equiv \frac{dx}{dt}\right)$ using Lagrange's equation is given by which of the following ?

(A) $x = t^4 + 2$

(B) $x = \frac{15}{7} t^3 + \frac{6}{7}$

(C) $x = 5t^2 - 2$

(D) $x = 5t^3 + 3$

89. Let ϕ satisfy $\phi(x) = f(x) + \int_0^x \sin(x-t)\,\phi(t)\,dt$. Then ϕ is given by :

(A) $\phi(x) = f(x) - \int_0^x \sin(x-t)\, f(t)\, dt$

(B) $\phi(x) = f(x) + \int_0^x (x-t)\, f(t)\, dt$

(C) $\phi(x) = f(x) - \int_0^x (x-t)\, f(t)\, dt$

(D) $\phi(x) = f(x) - \int_0^x \cos(x-t)\, f(t)\, dt$

90. Let $\phi(x)$ be the solution of $\int_0^x e^{x-t}\,\phi(t)\,dt = x,\ x > 0$. Then $\phi(1)$ equals :

(A) –1 (B) 0

(C) 1 (D) 2

91. A rigid body having one point O fixed and no external torque about O has equal principal moments of inertia. Then the body must rotate with :

(A) angular velocity with constant magnitude.

(B) angular velocity of variable magnitude.

(C) constant angular momentum but varying angular velocity.

(D) varying angular momentum with varying angular velocity.

92. Lagrange's equations for a Holonomic dynamical system specified by n-generalized coordinates q_j (j = 1, 2, 3….. n) having T the K.E. of system at time t and Q_j the generalized forces are :

(A) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=Q_j$

(B) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j$

(C) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=\dot{Q}_j$

(D) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)+\frac{\partial T}{\partial q_j}=\dot{Q}_j$

93. Let q_i and $\dot{q}_i$ respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are :

(A) $\dot{q}_i=\frac{\partial H}{\partial p_i},\ \dot{p}_i=\frac{\partial H}{\partial q_i}$

(B) $\dot{q}_i=\frac{\partial H}{\partial p_i},\ \dot{p}_i=-\frac{\partial H}{\partial q_i}$

(C) $\dot{q}_i=-\frac{\partial H}{\partial p_i},\ \dot{p}_i=\frac{\partial H}{\partial q_i}$

(D) $\dot{q}_i=-\frac{\partial H}{\partial p_i},\ \dot{p}_i=-\frac{\partial H}{\partial q_i}$

94. There are two boxes. Box 1 contains 2 red balls and 4 green balls. Box 2 contains 4 red balls and 2 green balls. A box is selected at random and a ball is chosen randomly from the selected box. If the ball turns out to be red, what is the probability that Box 1 had been selected ?

(A) 1/2 (B) 1/6

(C) 2/3 (D) 1/3

95. Suppose customers arrive in a shop according to a Poisson process with rate 4 per hour. The shop opens at 10 : 00 am. If it is given that the second customer arrives at 10 : 40 am, what is the probability that no customer arrived before 10 : 30 am ?

(A) 1/4 (B) e^{-2}

(C) 1/2 (D) $e^{-1/2}$

96. Let $\psi(t)=e^{-|t|-\frac{t^2}{2}}$ and $\varphi(t)=\frac{e^{-|t|}+e^{-\frac{t^2}{2}}}{2}$. Which of the following is *true* ?

(A) ψ is a characteristic function but ϕ is not

(B) φ is a characteristic function but ψ is not

(C) both ψ and φ are characteristic functions

(D) neither ψ nor φ is a characteristic function

97. Consider the following linear programming problem : Maximize $z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \geq 1$; $x_1 + x_2 \leq 5$; $2x_1 - 3x_2 \leq 6$; $-2x_1 + 3x_2 \leq 6$. The problem has :

(A) an unbounded solution

(B) exactly one optimal solution

(C) more than one optimal solution

(D) no feasible solutions

98. A parallel system consists of n identical components. The lifetimes of the components are independent identically distributed uniform random variables with mean 30 hours and range 60 hours. If the expected lifetime of the system is 50 hours, then the value of n is :

(A) 3 (B) 4

(C) 5 (D) 6

99. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from uniform $(\theta, 5\theta)$, $\theta > 0$. Define $X_{(1)} = \min(X_1, X_2, \ldots\ldots, X_n)$ and $X_{(n)} = \max(X_1, X_2, \ldots\ldots, X_n)$. The Maximum likelihood estimator of θ is :

(A) $\frac{X_{(1)}}{5}$ (B) $X_{(n)}$

(C) $X_{(1)}$ (D) $\frac{X_{(n)}}{5}$

100. Suppose we draw a random sample of size n from a population of size N, where $1 < n < N$, using simple random sampling without replacement scheme. Let P be the population proportion of units possessing a particular attribute and p be the corresponding sample proportion. Which of the following is an unbiased estimator for P (1 – P) ?

(A) $\frac{n\,(N-1)}{N(n-1)}\,p(1-p)$

(B) $p(1-p)$

(C) $\frac{N-n}{N-1}\,p(1-p)$

(D) $\frac{N(n-1)}{n(N-1)}\,p(1-p)$